Algebra Round

20 minutes | 12 problems

1. Two stars are located on the coordinate plane at (3, 0) and (8, 12). What is the distance between them?

2. Maria can build 3 mini spaceships in 8 minutes. Victor can build 7 mini spaceships in 15 minutes. If they work together, how many complete mini spaceships can they build in 20 minutes?

3. The function f satisfies the relation $f(x + 3) + f(x - 1) = 4x + 10$ for all real x, and is used to compute stellar coordinates. What is $f(10)$?

4. A mining robot on the asteroid Ceres only activates if its distance of x satisfies the inequality $|2x - 15| < 7$. For how many integer values of x does the robot activate?

5. An astronaut, astronaut A, leaves his spaceship and heads for Planet A, which is $4x + 82$ meters away. His astronaut friend, astronaut B, also leaves from the same spaceship but instead heads for Planet B which is $12(x + 3)$ meters away. Once they have both reached their points, they realize that astronaut A has traveled twice the distance of astronaut B. The value of x can be expressed in simplest form as a fraction of $\frac{a}{b}$. What is $a + b$?

6. A probe that dove into Jupiter's atmosphere can be modeled by $f(t) = t^2-12t+5$, with t in seconds and f(t) measured in meters. How many meters deep does the probe dive into Jupiter?

7. A comet's position in space is described by $(t - 1,\ 3t + 5)$, where t is in days. For example, at time $t = 2$ days, the comet is at the coordinates $(1,\ 11)$. What is the speed of the comet in $\frac{unit}{day}$?

8. NASA's new satellite system carries two types of sensor modules. Thermal modules have a mass of x kg, while ion modules have a mass of y kg. During the first test, engineers load 5 thermal and 7 ion modules onto the satellite for a total mass of 1095 kg. Later, engineers remove 2 thermal modules and add 3 ion modules, and the total mass becomes 1,206 kg. What is the value of y?

9. Two astronauts, Peter and Bella, want to buy a few necessities for their intergalactic travels. Peter buys 2 Ready-To-Eat Meals and 1 bottle of water for $19.00. Bella's journey is longer,

so she buys 4 Ready-To-Eat Meals and 3 bottles of water for $42.00. What is the sum of 1 Ready-To-Eat Meal and 1 bottle of water?

10. Rebecca and Calvin are competing in a spaceship race. Because Calvin has a faster ship, he gives Rebecca a 5-second headstart. If the length of the race is 100 meters, and Rebecca's ship travels at a constant speed of 8 m/s, then how fast must Calvin's ship be for them to finish at the same time?

11. Adrian is traveling close to a black hole. Due to this, his movement in the coordinate plane is slightly different; if Adrian traveled d meters without interference from the black hole, he now travels 2^d meters. Adrian's position *with* interference from the black hole can be modeled as

$$p(t) = 6t^2 + 8t,$$

where p is his position in meters at time t. What would Adrian's position be *without* interference at time $t = 4$?

12. The amount of coolant needed for x gallons of fuel in Elon's rocketship can be modeled by

$$c(x) = \frac{x^8 - 1}{x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1},$$

Where $c(x)$ is the amount of coolant in gallons. How much coolant is needed if his rocketship has 13 gallons of fuel?